1.2 Analyzing Motion

The deafening roar of the engine of a competitor's tractor conveys the magnitude of the force that is applied to the sled in a tractor-pull contest. As the sled begins to move, weights shift to increase frictional forces. Despite the power of their engines, most tractors are slowed to a standstill before reaching the end of the 91 m track. In contrast to the brute strength of the tractors, dragsters "sprint" to the finish line. Many elements of the two situations are identical, however, since forces applied to masses change the linear (straight-line) motion of a vehicle.

In the previous section, you focussed on basic **dynamics** — the cause of changes in motion. In this section, you will analyze **kinematics** — the motion itself — in more detail. You will consider objects moving horizontally in straight lines.

SECTION EXPECTATIONS

- Analyze, predict, and explain linear motion of objects in horizontal planes.
- Analyze experimental data to determine the net force acting on an object and its resulting motion.

KEY TERMS

- dynamics
- kinematics
- uniform motion
- uniformly accelerated motion
- free-body diagram
- frictional forces
- coefficient of static friction
- coefficient of kinetic friction

Kinematic Equations

To analyze the motion of objects quantitatively, you will use the kinematic equations (or equations of motion) that you learned in previous courses. The two types of motion that you will analyze are **uniform motion** — motion with a constant velocity — and **uniformly accelerated motion** — motion under constant acceleration. When you use these equations, you will apply them to only one dimension at a time. Therefore, vector notations will not be necessary, because positive and negative signs are all that you will need to indicate direction. The kinematic equations are summarized on the next page, and apply only to the type of motion indicated.

Figure 1.5 In a tractor pull, vehicles develop up to 9000 horsepower to accelerate a sled, until they can no longer overcome the constantly increasing frictional forces. Dragsters, on the other hand, accelerate right up to the finish line.

Uniform motion

- definition of velocity $v = \dfrac{\Delta d}{\Delta t}$
- Solve for displacement in terms of velocity and time. $\Delta d = v\Delta t$

Uniformly accelerated motion

- definition of acceleration $a = \dfrac{\Delta v}{\Delta t}$ or $a = \dfrac{v_2 - v_1}{\Delta t}$
- Solve for final velocity in terms of initial velocity, acceleration, and time interval. $v_2 = v_1 + a\Delta t$
- displacement in terms of initial velocity, final velocity, and time interval $\Delta d = \dfrac{(v_1 + v_2)}{2}\Delta t$
- displacement in terms of initial velocity, acceleration, and time interval $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$
- final velocity in terms of initial velocity, acceleration, and displacement $v_2^2 = v_1^2 + 2a\Delta d$

Conceptual Problem

- The equations above are the most fundamental kinematic equations. You can derive many more equations by making combinations of the above equations. For example, it is sometimes useful to use the relationship $\Delta d = v_2\Delta t - \frac{1}{2}a\Delta t^2$. Derive this equation by manipulating two or more of the equations above. (Hint: Notice that the equation you need to derive is very similar to one of the equations in the list, with the exception that it has the final velocity instead of the initial velocity. What other equation can you use to eliminate the initial velocity from the equation that is similar to the desired equation?)

Combining Dynamics and Kinematics

When analyzing motion, you often need to solve a problem in two steps. You might have information about the forces acting on an object, which you would use to find the acceleration. In the next step, you would use the acceleration that you determined in order to calculate some other property of the motion. In other cases, you might analyze the motion to find the acceleration and then use the acceleration to calculate the force applied to a mass. The following sample problem will illustrate this process.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of acceleration and velocity.

SAMPLE PROBLEM

Finding Velocity from Dynamics Data

In television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass 9.1×10^{-31} kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Conceptualize the Problem

- The electrons are moving *horizontally*, from the back to the front of the tube, under an *electric force*.
- The *force of gravity* on an electron is exceedingly small, due to the electron's small mass. Since the electrons move so quickly, the time interval of the entire flight is very short. Therefore, the *effect of the force of gravity is too small to be detected* and you can consider the electric force to be the only force affecting the electrons.
- Information about *dynamics data* allows you to find the electrons' *acceleration*.
- Each electron is *initially at rest*, meaning that the *initial velocity is zero*.
- Given the acceleration, the equations of motion lead to other variables of motion.
- Let the direction of the force, and therefore the direction of the acceleration, be positive.

Identify the Goal

The final velocity, v_2, of an electron when exiting the electron gun

Identify the Variables and Constants

Known	Implied	Unknown
$m_e = 9.1 \times 10^{-31}$ kg	$v_1 = 0\ \frac{\text{m}}{\text{s}}$	a
$F = 5.8 \times 10^{-15}$ N		v_2
$\Delta d = 3.5 \times 10^{-3}$ m		

Develop a Strategy

Apply Newton's second law to find the net force.

$$\vec{F} = m\vec{a}$$

Write Newton's second law in terms of acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$

Substitute and solve.

$$\vec{a} = \frac{+5.8 \times 10^{-15}\ \text{N}}{9.1 \times 10^{-31}\ \text{kg}}$$

$\frac{\text{N}}{\text{kg}}$ is equivalent to $\frac{\text{m}}{\text{s}^2}$.

$$\vec{a} = 6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\text{[toward the front of tube]}$$

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Apply the kinematic equation that relates initial velocity, acceleration, and displacement to final velocity.

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2^2 = 0 + 2\left(6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\right)(3.5 \times 10^{-3}\ \text{m})$$
$$v_2 = 6.67\ 967 \times 10^6\ \frac{\text{m}}{\text{s}}$$
$$v_2 \cong 6.7 \times 10^6\ \frac{\text{m}}{\text{s}}$$

The final velocity of the electrons is about 6.7×10^6 m/s in the direction of the applied force.

Validate the Solution

Electrons, with their very small inertial mass, could be expected to reach high speeds. You can also solve the problem using the concepts of work and energy that you learned in previous courses. The work done on the electrons was converted into kinetic energy, so $W = F\Delta d = \frac{1}{2}mv^2$. Therefore,

$$v = \sqrt{\frac{2F\Delta d}{m}} = \sqrt{\frac{2(5.8 \times 10^{-15}\ \text{N})(3.5 \times 10^{-3}\ \text{m})}{9.1 \times 10^{-31}\ \text{kg}}} = 6.679 \times 10^6\ \frac{\text{m}}{\text{s}} \cong 6.7 \times 10^6\ \frac{\text{m}}{\text{s}}.$$

Obtaining the same answer by two different methods is a strong validation of the results.

PRACTICE PROBLEMS

1. A linear accelerator accelerated a germanium ion ($m = 7.2 \times 10^{-25}$ kg) from rest to a velocity of 7.3×10^6 m/s over a time interval of 5.5×10^{-6} s. What was the magnitude of the force that was required to accelerate the ion?
2. A hockey stick exerts an average force of 39 N on a 0.20 kg hockey puck over a displacement of 0.22 m. If the hockey puck started from rest, what is the final velocity of the puck? Assume that the friction between the puck and the ice is negligible.

Determining the Net Force

In almost every instance of motion, more than one force is acting on the object of interest. To apply Newton's second law, you need to find the resultant force. A free-body diagram is an excellent tool that will help to ensure that you have correctly identified and combined the forces.

To draw a **free-body diagram**, start with a dot that represents the object of interest. Then draw one vector to represent each force acting on the object. The tails of the vector arrows should all start at the dot and indicate the direction of the force, with the arrow-head pointing away from the dot. Study Figure 1.6 to see how a free-body diagram is constructed. Figure 1.6 (A) illustrates a crate being pulled across a floor by a rope attached to the edge of the crate. Figure 1.6 (B) is a free-body diagram representing the forces acting on the crate.

Two of the most common types of forces that influence the motion of familiar objects are frictional forces and the force of gravity. You will probably recall from previous studies that the

magnitude of the force of gravity acting on objects on or near Earth's surface can be expressed as $F = mg$, where g (which is often called the acceleration due to gravity) has a value 9.81 m/s^2. Near Earth's surface, the force of gravity always points toward the centre of Earth.

Whenever two surfaces are in contact, **frictional forces** oppose any motion between them. Therefore, the direction of the frictional force is always opposite to the direction of the motion. You might recall from previous studies that the magnitudes of frictional forces can be calculated by using the equation $F_f = \mu F_N$. The normal force in this relationship (F_N) is the force perpendicular to the surfaces in contact. You might think of the normal force as the force that is pressing the two surfaces together. The nature of the surfaces and their relative motion determines the value of the coefficient of friction (μ). These values must be determined experimentally. Some typical values are listed in Table 1.1.

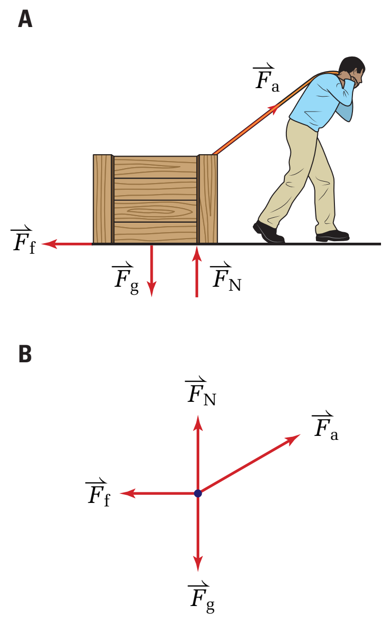


Figure 1.6 **(A)** The forces of gravity ($\vec{F}_g$), friction ($\vec{F}_f$), the normal force of the floor ($\vec{F}_N$), and the applied force of the rope ($\vec{F}_a$) all act on the crate at the same time. **(B)** The free-body diagram includes *only* those forces acting *on* the crate and none of the forces that the crate exerts on other objects.

Table 1.1 Coefficients of Friction for Some Common Surfaces

Surface	Coefficient of static friction (μ_s)	Coefficient of kinetic friction (μ_k)
rubber on dry, solid surfaces	1–4	1
rubber on dry concrete	1.00	0.80
rubber on wet concrete	0.70	0.50
glass on glass	0.94	0.40
steel on steel (unlubricated)	0.74	0.57
steel on steel (lubricated)	0.15	0.06
wood on wood	0.40	0.20
ice on ice	0.10	0.03
Teflon™ on steel in air	0.04	0.04
ball bearings (lubricated)	<0.01	<0.01
joint in humans	0.01	0.003

If the objects are not moving relative to each other, you would use the **coefficient of static friction** (μ_s). If the objects are moving, the somewhat smaller **coefficient of kinetic friction** (μ_k) applies to the motion.

As you begin to solve problems involving several forces, you will be working in one dimension at a time. You will select a coordinate system and resolve the forces into their components in each dimension. Note that the components of a force are not vectors themselves. Positive and negative signs completely describe the motion in one dimension. Thus, when you apply Newton's laws to the components of the forces in one dimension, you will not use vector notations.

ELECTRONIC LEARNING PARTNER

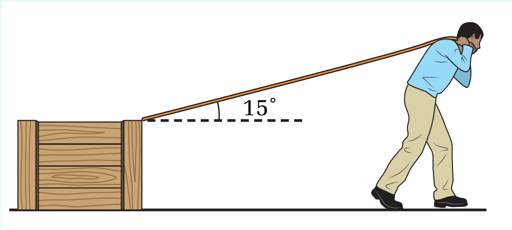
Refer to your Electronic Learning Partner to enhance your understanding of forces and vectors.

Another convention used in this textbook involves writing the sum of all of the forces in one dimension. In the first step, when the forces are identified as, for example, gravitational, frictional, or applied, only plus signs will be used. Then, when information about that specific force is inserted into the calculation, a positive or negative sign will be included to indicate the direction of that specific force. Watch for these conventions in sample problems.

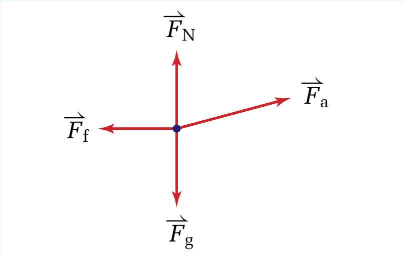
SAMPLE PROBLEM

Working with Three Forces

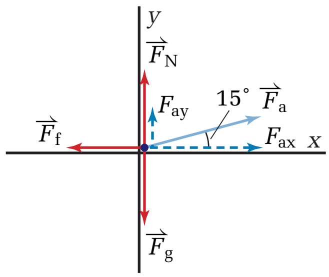
To move a 45 kg wooden crate across a wooden floor ($\mu = 0.20$), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of 115 N, it makes an angle of 15° with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of 1.4 m/s?



Conceptualize the Problem



- To start framing this problem, draw a free-body diagram.
- *Motion* is in the horizontal direction, so the net *horizontal force* is causing the crate to *accelerate*.
- Let the *direction* of the motion be the *positive* horizontal direction.
- There is no motion in the vertical direction, so the *vertical acceleration is zero*. If the acceleration is zero, the *net vertical force* must be *zero*. This information leads to the value of the *normal force*. Let "up" be the *positive* vertical direction.
- Since the beginning of the time interval in question is the instant at which the crate begins to move, the *coefficient of kinetic friction* applies to the motion.
- Once the *acceleration* is found, the *kinematic equations* allow you to determine the values of other quantities involved in the motion.



Identify the Goal

The time, Δt, required to reach a velocity of 1.4 m/s

Identify the Variables

Known		Implied	Unknown	
$\vec{F}_a = +115$ N	$m = 45$ kg	$v_i = 0\ \frac{m}{s}$	$\vec{F}_N$	$\vec{a}$
$\theta = 15°$	$v_f = 1.4\ \frac{m}{s}$	$g = 9.81\ \frac{m}{s^2}$	$\vec{F}_g$	Δt
$\mu = 0.20$			$\vec{F}_f$	

Develop a Strategy

To find the normal force, apply Newton's second law to the vertical forces. Analyze the free-body diagram to find all of the vertical forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(\text{vertical})} + F_g + F_N = ma$$
$$F_g = -mg$$
$$F_{a(\text{vertical})} - mg + F_N = ma$$
$$F_N = ma + mg - F_{a(\text{vertical})}$$
$$F_N = 0 + (45\text{ kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right) - (115\text{ N})\sin 15^\circ$$
$$F_N = 441.45\text{ N} - 29.76\text{ N}$$
$$F_N = 411.69\text{ N}$$

To find the acceleration, apply Newton's second law to the horizontal forces. Analyze the free-body diagram to find all of the horizontal forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(\text{horizontal})} + F_f = ma$$
$$F_f = -\mu F_N$$
$$a = \frac{F_{a(\text{horizontal})} - \mu F_N}{m}$$
$$a = \frac{(115\text{ N})\cos 15^\circ - (0.20)(411.69\text{ N})}{45\text{ kg}}$$
$$a = \frac{111.08\text{ N} - 82.34\text{ N}}{45\text{ kg}}$$
$$a = 0.6387\,\frac{\text{m}}{\text{s}^2}$$

To find the time interval, use the kinematic equation that relates acceleration, initial velocity, final velocity, and time.

$$a = \frac{v_f - v_i}{\Delta t}$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$\Delta t = \frac{1.4\,\frac{\text{m}}{\text{s}} - 0\,\frac{\text{m}}{\text{s}}}{0.6387\,\frac{\text{m}}{\text{s}^2}}$$
$$\Delta t = 2.19\text{ s}$$
$$\Delta t \cong 2.2\text{ s}$$

You will be pulling the crate at 1.4 m/s at 2.2 s after the crate begins to move.

Validate the Solution

Check the units for acceleration: $\frac{\text{N}}{\text{kg}} = \frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}}{\cancel{\text{kg}}} = \frac{\text{m}}{\text{s}^2}$. The units are correct. A velocity of 1.4 m/s is not very fast, so you would expect that the time interval required to reach that velocity would be short. The answer of 2.2 s is very reasonable.

PRACTICE PROBLEMS

3. In a tractor-pull competition, a tractor applies a force of 1.3 kN to the sled, which has mass 1.1×10^4 kg. At that point, the coefficient of kinetic friction between the sled and the ground has increased to 0.80. What is the acceleration of the sled? Explain the significance of the sign of the acceleration.

4. A curling stone with mass 20.0 kg leaves the curler's hand at a speed of 0.885 m/s. It slides 31.5 m down the rink before coming to rest.

(a) Find the average force of friction acting on the stone.

(b) Find the coefficient of kinetic friction between the ice and the stone.

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